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Manifolds and Group actions

Homework 5

Mandatory Exercise 1. (5 Points)
Let $n \in \mathbb{N}$ and let $S^{2n-1} \subset \mathbb{R}^{2n}$ be the odd dimensional sphere. Construct a non-vanishing vector field on the odd dimensional sphere.

Mandatory Exercise 2. (10 points)
Let $F : M \to N$ be a diffeomorphism. Let φ_t be the flow of a complete vector field X on M. Compute the flow of $F_*(X)$.

Mandatory Exercise 3. (5 points)
Let X, Y, Z be vector fields on M and $f : M \to \mathbb{R}$ a function. Prove the following identities

$$[X, fY] = X(f)Y + f[X, Y].$$

$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0.$$

Suggested Exercise 1. (0 Points)
Let X, Y and Z be vectorfields on $\mathbb{R}^2$ given by

$$X_{(x,y)} = \frac{\partial}{\partial x}, \quad Y_{(x,y)} = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}, \quad Z_{(x,y)} = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}.$$

a) Compute the flows of X, Y, Z and investigate its properties. Check that the flows of Y and Z commute.

b) Compute the Lie brackets $[X, Y]$ and $[Y, Z]$ and discuss their flows.

Suggested Exercise 2. (0 points)
Show that the vector field X on $\mathbb{R}$ given by

$$X_x = x^2\frac{\partial}{\partial x},$$

is not complete.

Hand in on Monday 22 of May in the pigeonhole on the third floor.